# Modeling the Gate I/V Characteristic of a GaAs MESFET for Volterra-Series Analysis

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## PREFACE

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## I. INTRODUCTION

Volterra-series analysis 1-3 is an efficient and practical method for determining intermodulation levels in small-signal MESFET amplifiers. The Volterra series is particularly valuable when implemented in a general-purpose computer program, because it can be used to analyze very large or complex circuits, it does not require that the circuit topology be simplified, and it is significantly more efficient than time-domain or harmonic-balance methods.

Although much research has been directed at developing large-signal FET models for use with harmonic-balance analysis, very little work has been done to model GaAs MESFETs for Volterra-series analysis. Modeling GaAs MESFETS presents a number of subtle problems that occasionally have been noted by other authors (e.g. Refs. 5, 6); one of the most serious of these problems is the difficulty of modeling the FET's "nonlinear transconductance," or more precisely its nonlinear incremental gate/drain I/V characteristic. This report describes a method for determining the parameters of this characteristic by means of simple, low-frequency rf measurements.

#### II. THE MESFET MODEL

Figure 1 shows a small signal nonlinear model of a MESFET. This model consists of a modified conventional small-signal linear model, in which three elements — the gate/source capacitance  $C_{gs}$ , the drain/source resistance  $R_{ds}$ , and the drain current  $i_d(v_g)$  — are nonlinear. Although  $C_{gs}$  and  $R_{ds}$  are significant sources of intermodulation in small-signal FETs, in most cases the dominant nonlinearity is that of  $i_d(v_g)$ .

The nonlinear resistive elements are characterized by Taylor-series expansions of their I/V characteristics in the vicinity of the dc bias point. Thus, the nonlinearity  $i_d(v_g)$  is

$$i_{d} = \frac{dI_{d}(v_{g})}{dv_{g}} \bigg| v_{g} + \frac{1}{2} \frac{d^{2}I_{d}(v_{g})}{dv_{g}^{2}} \bigg| v_{g}^{2} + \frac{1}{6} \frac{d^{3}I_{d}(v_{g})}{dv_{g}^{3}} \bigg| v_{g}^{3} + \dots$$

$$v_{g} = v_{g,0} \qquad v_{g} = v_{g,0} \qquad v_{g} = v_{g,0} \qquad (1)$$

where  $I_d(V_g)$  is the large-signal gate/drain characteristic and  $i_d$  and  $v_g$  are, respectively, the incremental (rf) drain and gate currents and voltages, i.e., the current and voltage deviations from the bias point  $I_d(V_{g,0})$ . Then  $i_d$  is expressed as

$$i_d = a_1 v_g + a_2 v_g^2 + a_3 v_g^3 + \dots$$
 (2)

The traditional method of determining the series coefficients  $\mathbf{a}_n$  is to measure the I/V characteristic at a fixed drain-bias voltage and to perform a least-squares fit of  $\mathbf{i}_{\mathbf{d}}(\mathbf{v}_{\mathbf{g}})$  to a polynomial of the desired degree. Although this process is usually adequate for determining  $\mathbf{a}_1$  (which is equivalent to the linear transconductance), it is often unsatisfactory for determining the higher-order coefficients: because of the ill-conditioned nature of the normal equation, the values of  $\mathbf{a}_n$  determined in this manner are very sensitive to measurement inaccuracy, round-off errors, and the selection of data points.

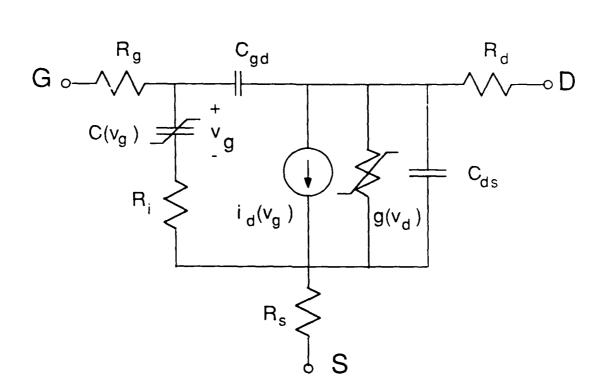


Fig. 1. Small-Signal, Nonlinear Equivalent Circuit of a GaAs MESFET

Differentiating the measured data directly is often adequate for determining  $a_1$  and sometimes  $a_2$ ; however, differentiation exacerbates the effects of round-off errors and thus makes the higher derivatives unreliable.

A more fundamental problem is related to traps in the FET's channel; these introduce long time constants into  $i_d(v_g)$  and cause differences between the dc and rf I/V characteristics. Trapping effects are often not evident in  $a_1$  or  $a_2$ , but derivatives of higher order are very sensitive to them. This is particularly the case when automated equipment is used to measure  $I_d(V_g)$ ; differences in stepping speed and dwell time at each data point affect the coefficients' values strongly.

An effective way of circumventing these problems is to derive the series coefficients from rf measurements instead of dc. The rf measurements are made at a frequency in the VHF range, at which both trapping effects and the FET's reactive parasitics are negligible. Under these conditions the levels of the harmonic output components are functions of only the input power, the FET's source resistance, and the series coefficients. It is a simple matter to measure the harmonic levels and to derive the coefficients from them.

#### III. DESCRIPTION OF THE MEASUREMENTS

When the rf frequency is very low, the equivalent circuit of Figure 1 can be simplified to form the circuit shown in Fig. 2. In Fig. 2 the FET is driven by an rf source at a frequency of approximately 50 MHz, and the drain is terminated in a load resistance  $R_L$ , where  $R_L << R_{\rm ds}$ .  $R_L$  is a shunting resistor that reduces the current in  $R_{\rm ds}$  to the point where its effects are negligible; one should make  $R_L$  as small as possible while still allowing harmonic output components to be observable on a spectrum analyzer. Because the series coefficients are derived from the relative levels of the harmonics, and not their absolute powers, the value of  $R_L$  does not affect the accuracy of the measurement.

Using the method of nonlinear currents,  $^{2,3}$  one can show that the first three harmonics of the drain current,  $I_{1-3}$ , are

$$I_{1} = \frac{a_{1}V_{gs}}{1 + a_{1}R_{s}} \tag{3}$$

$$I_2 = \frac{a_2(1 - a_1 R_s) V_{gs}^2}{2(1 + a_1 R_s)^2}$$
 (4)

$$I_{3} = \frac{(a_{3} - 2a_{2}^{2}R_{s})(1 - a_{1}R_{s})V_{gs}^{3}}{4(1 + a_{1}R_{s}^{3})}$$
(5)

 $V_{gs}$  is the magnitude of  $v_{gs}(t)$ ; i.e.,

$$v_{gs}(t) = V_{gs} \cos(\omega t) \tag{6}$$

and the available power  $\mathbf{P}_{\mathbf{a}}$  of the source is

$$P_{a} = \frac{V_{gs}^{2}}{8 R_{in}} \tag{7}$$

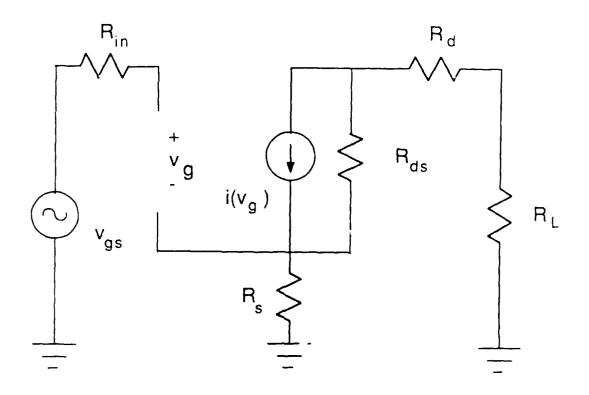


Fig. 2. FET-Equivalent Circuit Valid at Frequencies in the VHF Range.  $\rm R_L$  is the output-shunting resistor used to minimize harmonic generation in  $\rm R_{ds}$  .

where  $R_{in}$  is the source resistance.

Substituting Eq. (7) into Eqs. (3) and (4), dividing Eq. (4) by Eq. (3), and squaring the result gives the ratio of second-harmonic output power to fundamental output power,  $IMR_2$ :

$$IMR_{2} = \frac{2 a_{2}^{2} R_{in} (1 - a_{1} R_{2})^{2} P_{a}}{a_{1}^{2} (1 + a_{1} R_{s})^{2}}$$
(8)

A similar process gives the ratio of third-harmonic output power to fundamental output power,  $IMR_3$ :

$$IMR_{3} = \left[\frac{2(a_{3} - 2a_{2}^{2}R_{s})(1 - a_{1}R_{s})R_{in}}{a_{1}(1 + a_{1}R_{s})^{2}}\right]P_{a}^{2}$$
 (9)

When the FET is biased and excited by a low-frequency ri source, the values of  $a_n$  can be found by measuring the harmonic output power and solving Eqs. (7) through (9). The process is as follows:

- 1. a<sub>1</sub> is determined in the conventional manner, i.e., by dc or rf measurements.
- 2. IMR<sub>2</sub> is measured by means of a spectrum analyzer, and Eq. (8) is solved to determine  $a_2$ .
- 3. IMR<sub>3</sub> is measured similarly, and Eq. (9) is solved to determine  $a_3$ .

Equation (8) does not indicate whether  $a_2$  is positive or negative. However, because a MESFET's transconductance invariably rises with  $v_g$ ,  $a_2$  is invariably positive (this is generally not the case in HEMTs, but even then the regions of positive and negative  $a_2$  are clear from the transconductance curve). Because of the squared term in brackets, two values of  $a_3$  satisfy Eq. (9). One can determine the correct root by measuring a few values of  $a_2$  at nearby bias points and picking the value of  $a_3$  that most closely matches  $a_3 = 0.33 \; da_2/dv_g$ . Usually the two values of  $a_3$  have different signs; then, to select the correct value of  $a_3$ , one need determine only whether  $a_2$  rises or falls with  $v_g$ .

#### IV. EXPERIMENTAL RESULTS

This method was used to determine the incremental gate I/V characteristic of a packaged Avantek AT10650-5 MESFET biased at a drain voltage of 3 V and drain current of 20 mA. The FET's transconductance was measured at dc, and its small-signal equivalent circuit (including the package parasitics) was determined by adjusting its circuit-element values until good agreement between calculated and measured S parameters was obtained. The FET was then installed in a low-frequency test fixture having a shunting resistance  $\rm R_L$  of 5 ohms. The input power  $\rm P_a$  was -11 dBm at a frequency of 50 MHz.

The  $a_n$  coefficients were determined at 19, 20, and 21 mA, and the values in Table 1 were obtained. The coefficient  $a_2$  was found to decrease with  $I_d$ , and the two roots of Eq. (9) had different signs; therefore the negative value of  $a_3$  is the correct one.

Table 1. Taylor-Series Coefficients for the Avantek AT10650-5

Id	a <sub>1</sub>	a <sub>2</sub>	a <sub>3</sub>
0.019	0.041	0.0188	-0.0148
0.020	0.041	0.0171	-0.0145
0.021	0.041	0.0158	-0.0128

The FET's gate/source capacitance was modeled as an ideal Schottky-barrier junction capacitance, and the nonlinear drain/source resistance was derived from low-frequency Y parameters measured at a variety of bias voltages. When the source and load terminations were 50 ohms, the program described in Ref. 4 predicted the FET's third-order intermodulation intercept point to be 20.8 dBm at 10 GHz. This result

agrees well with the measured value of the intercept point, which was 22.4 dBm. When the output port was conjugate matched, the calculated intercept point was 24.1 dBm, compared to a measured value of 23.0 dBm. Other tests have shown equal or superior agreement. These results imply that the intermodulation levels in small-signal amplifiers can be predicted within 2 to 3 dB. We believe that the major factor limiting accuracy is the linear part of the FET's equivalent-circuit model, especially the part that models the package. Thus, even better accuracy could be obtained with chip or MMIC circuits.

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